# Fully Automated Attractor Analysis of Cyanobacteria Models

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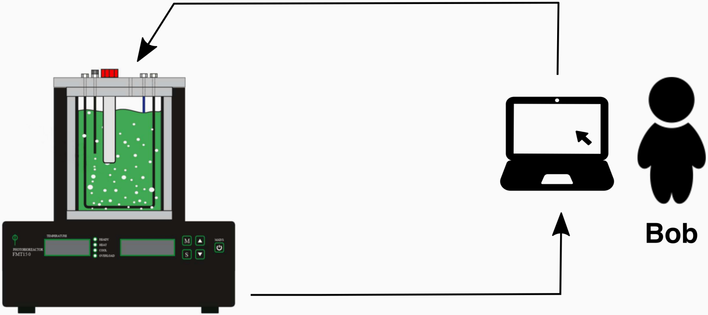

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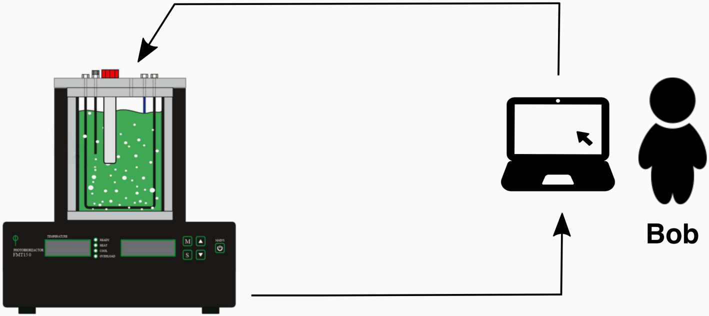


Model-based control ⇒ `e-cyanobacterium.org`

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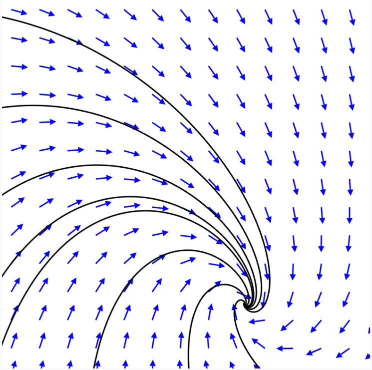
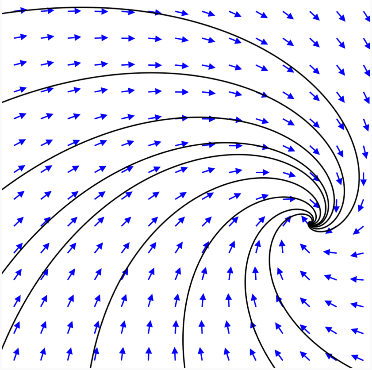


$p = 1.0$



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- Requires a skilled model analyst
- Computationally intensive, but hard to parallelise

# Method: Terminal strongly connected components

$$\frac{dx_1}{dt} = f_1(x_1, \ldots, x_n)$$
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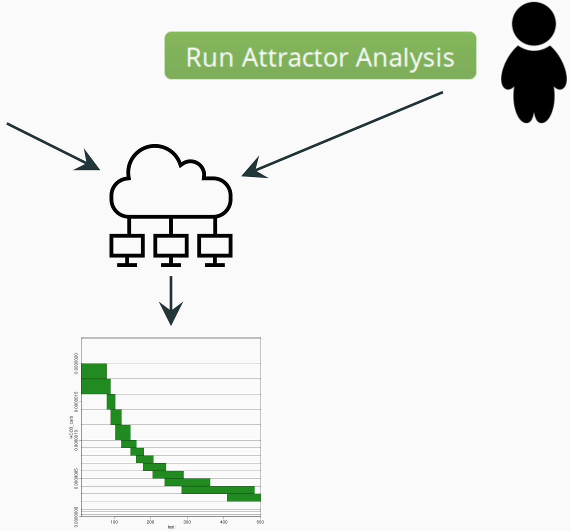
$$\frac{dx_n}{dt} = f_n(x_1, \ldots, x_n)$$



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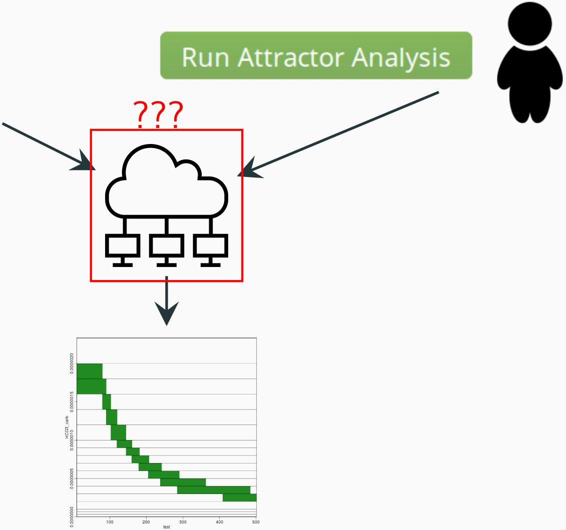

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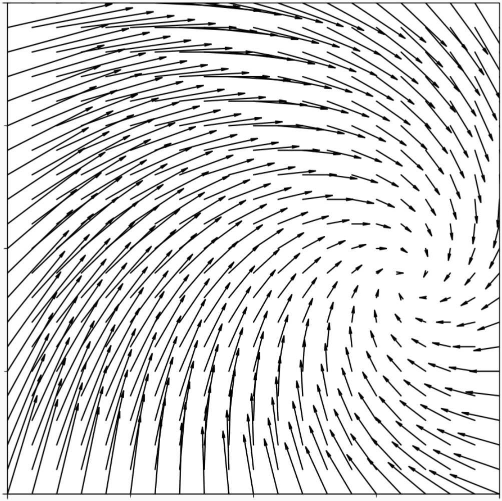

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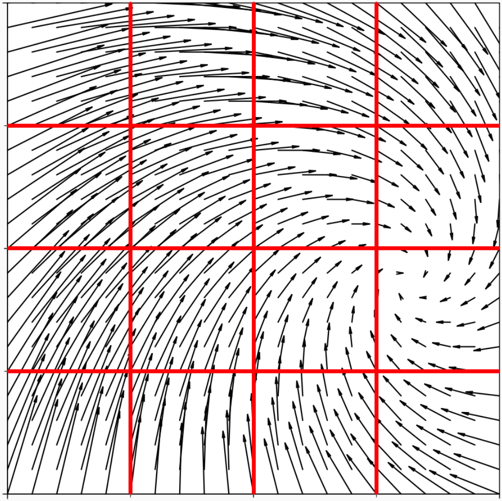

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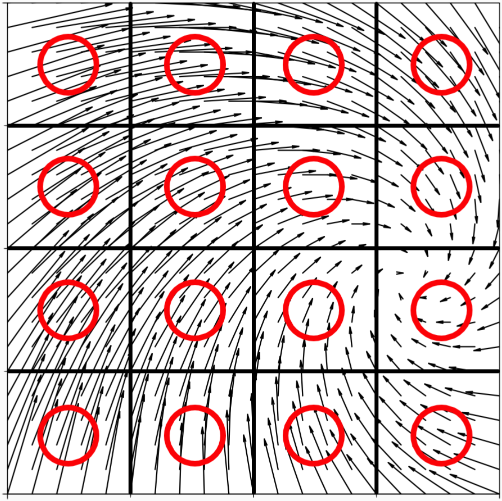

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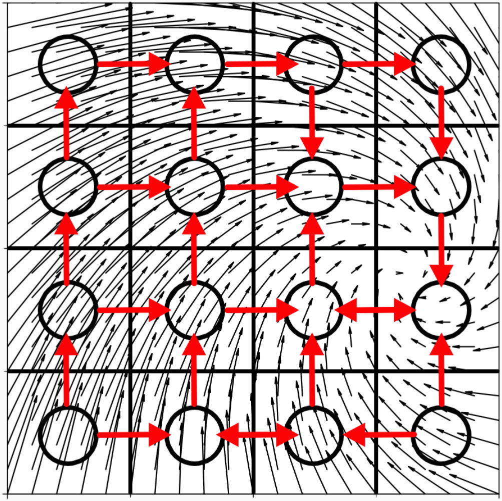

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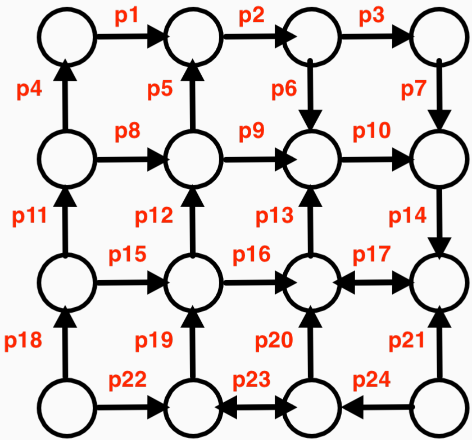

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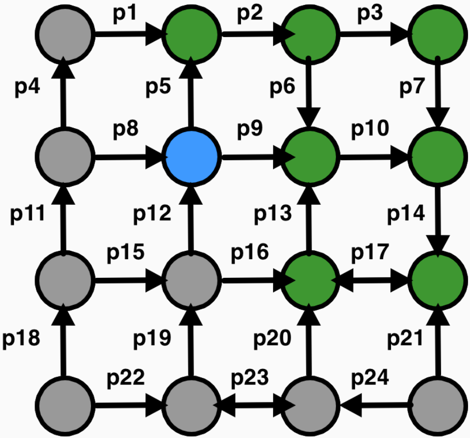

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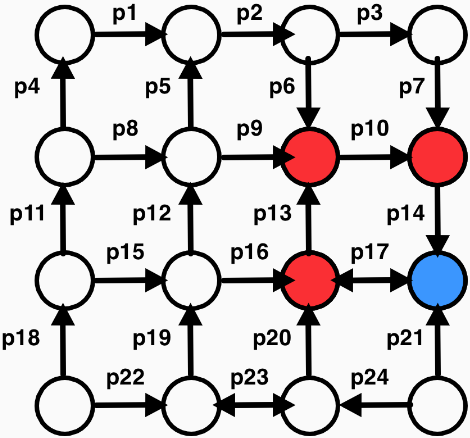

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3. Parallel parametrised divide and conquer algorithm for component detection
4. Each terminal component over-approximates an attractor

# Models

**Clark et al. 2014**

- Fluxes of inorganic carbon from cytosol to carboxysome
- Fixation using carbonic anhydrase and *RuBisCO* enzyme

**Grimaud et al. 2014**

- Time-dependent dynamics of nitrogen fixation
- Respecting the obligate nitrogen fixation and light limitation

**Müller et al. (in devel.)**

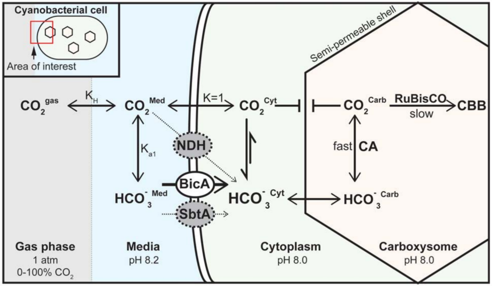
- Carbon fluxes in a laboratory scale photobioreactor
- Intercellular exchange, carbonate chemistry, and gas-to-liquid $CO_2$ transfer

**Plyusnina et al. (in devel.)**

- Electron transport on thylakoid membrane (photosynthesis)

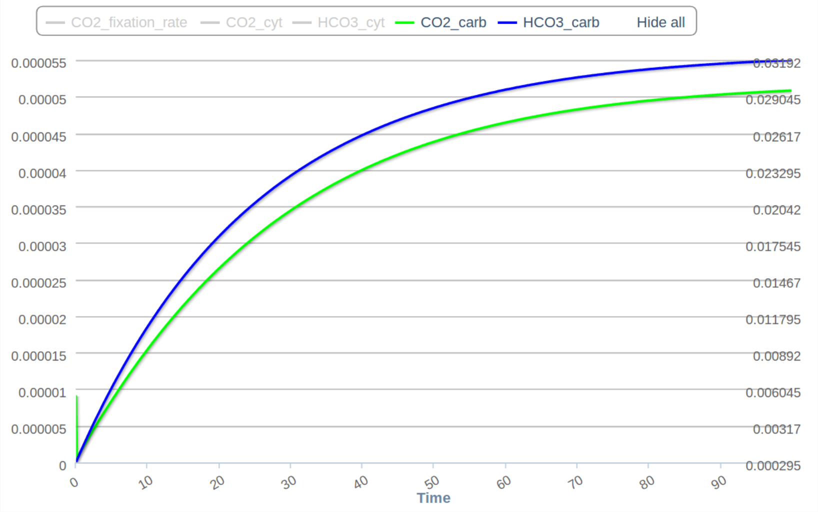
# Clark et al. 2014[1]

- Fluxes and fixation of inorganic carbon
- Carbon dioxide concentrating mechanism (CCM)
- Model shows that CCM is not necessary for growth in media in equilibrium with concentration of 10% $CO_2$
- Activity of carbon-fixing enzyme *RuBisCO*
- Parameter *fast* affects rate of carbon fixation reaction




[1] Ryan L. Clark et al., Insights into the industrial growth of cyanobacteria from a model of the carbon-concentrating mechanism, AIChE Journal, 2014, https://doi.org/10.1002/aic.14310

# Clark et al. – simulation

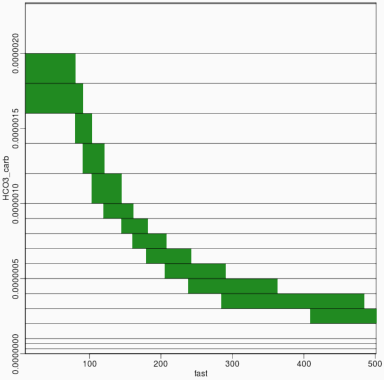


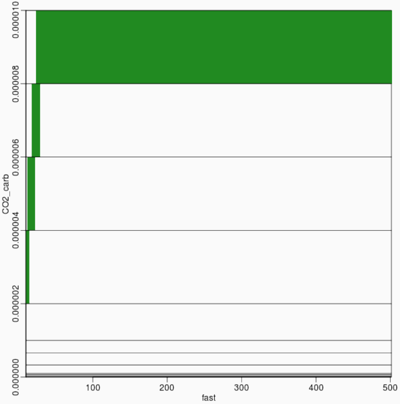

Parameter *fast* $= 100$

# Clark et al. – results

A single attractor across whole parameter range

$CO_2$ increases rapidly with *fast*, $HCO_3$ decreases for higher values



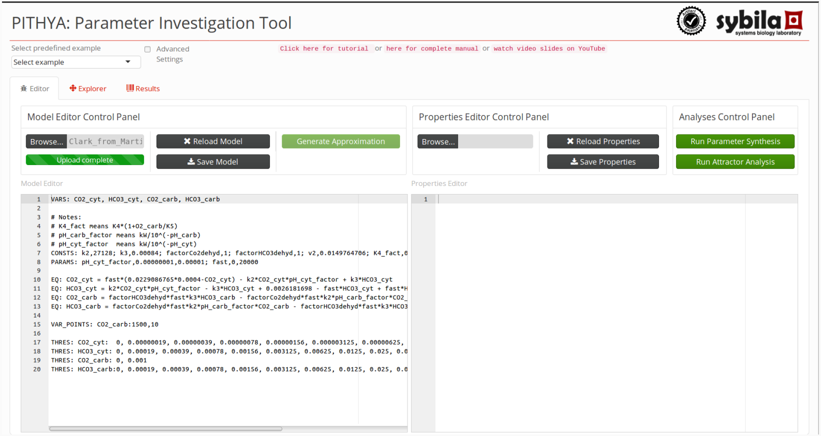



# Other results

- Clark: Strong dependence on parameter *fast*, 4 dimensions
- Grimaud: Independent on parameters $r_2$ and $r_4$, 4 dimensions
- Müller: Independent on parameter $kLA\_CO_2\_eff$, 7 dimensions
- Plyusnina: 8 dimensions, strange non-trivial attractor

# Software support: Pithya

http://pithya.ics.muni.cz/app/pithya



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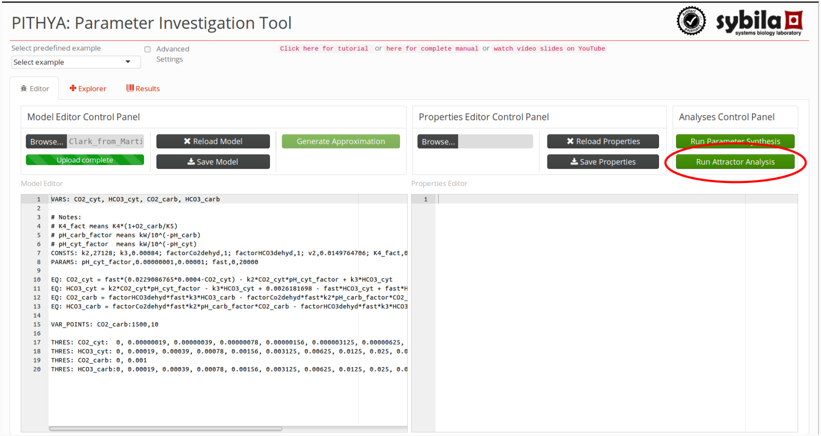

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Thank you for your attention!